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A NEW METHOD FOR RECOGNIZING QUADRIC SURFACES FROM RANGE DATA AND ITS APPLICATION TO TELEROBOTICS AND AUTOMATION

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A New Method for Recognizing Quadric Surfaces from Range Data and Its Application to Telerobotics and Automation (phase II)

by

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Abstract

Pose and orientation of an object is one of the central issues in 3-D recognition problems. Most of today's available techniques require considerable pre-processing, such as detecting edges or joints, fitting curves or surfaces to segment images and trying to extract higher order features from the input images. In this research We present a method based on analytical geometry, whereby all the rotation parameters of any quadric surface are determined and subsequently eliminated. This procedure is iterative in nature and has been found to converge to the desired results in as few as three iterations. The approach enables us to position the quadric surface in a desired coordinate system, then, utilize the presented shape information to explicitly represent and recognize the 3-D surface. Experiments were conducted with simulated data for objects such as hyperboloid of one and two sheets, elliptic and hyperbolic paraboloid, elliptic and hyperbolic cylinders, ellipsoids, and quadric cones. Real data of quadric cones and cylinders were also utilized. Both of theses sets yielded excellent results.

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1. INTRODUCTION

One of the most important tasks in computer vision is that of 3-D object recognition. Success has been limited to the recognition of symmetric objects and now researchers are investigating recognition of several asymmetric objects as well objects placed in complex scenes. Unlike the recognition procedure developed for intensity based images, the recent upsurge of several active and passive sensors extracting quality range information, has lead to the involvement of explicit geometric shapes of the objects for the recognition schemes [1]. Location and description of 3-D objects from natural light images are often difficult to obtain. Range images on the other hand give a more detailed and direct geometric description of the shape of the 3-D object.

The determination of the location and orientation of a 3-D object is one of the central problems in computer vision applications. It is observed that most of the methods and techniques which look into this problem require considerable preprocessing such as detecting edges or junctions, fitting curves or surfaces to segmented images and computing high order features from the input images. Since 3-D object recognition depends not only on the shape of the object but also the pose and orientation of the object as well, any definite information about the object's orientation will aid in selecting the right features for the recognition process.

In this research we put forward a method based on analytical geometry whereby all the rotation parameters of any object placed in any orientation in space are determined and eliminated systematically. With this approach we are in a position to place the 3-D object in a desired stable position thereby eliminating the orientation problem and subsequently utilize the shape information to explicitly represent the 3-D surface.

In the initial part of the research we discuss the rotation transformations and in the later part of the research we propose our scheme to eliminate the rotation parameters.

2. BACKGROUND

Most of the available techniques for describing and recognizing 3-D objects are based on the principle of segmentation. Segmentation is the process in which range data [1] is divided into smaller regions (mostly squares). These small regions are then approximated by planar surfaces or curved surfaces based upon the surface mean and gaussian curvatures. Regions sharing similar curvatures are subsequently merged, the process known as region growing. There are several other approaches [2,3,4,5,6] wherein the 3-D recognition problem has been dealt with. Levine et al. [7] briefly review the various works in the field of segmentation, whereby segmentation has been classified as region-based and edge-based approaches. Again surface curvatures play an important role while characterizing each of these approaches.

We have proposed an approach [8] based on two-dimensional analytic geometry to recognize a series of three-dimensional objects. An effective technique to determine the 3-D object location and orientation will aid us in extending the proposed scheme to various 3-D objects such as the hyperboloids of one and two sheets, the circular and elliptical quadric cones, the circular and elliptical cylinder, the parabolic and hyperbolic cylinders, the elliptic and hyperbolic paraboloids and so on. Figure (1) illustrates the various 3-D surfaces we propose to use for the recognition scheme. In our proposed method [8] a feature vector consisting of various 2-D curves obtained after intersection of objects with planes in various orientations serve as the medium of distinguishing objects from one another. Sets of range data of objects are attempted to be identified as quadric surfaces based upon their representation by a second degree polynomial.

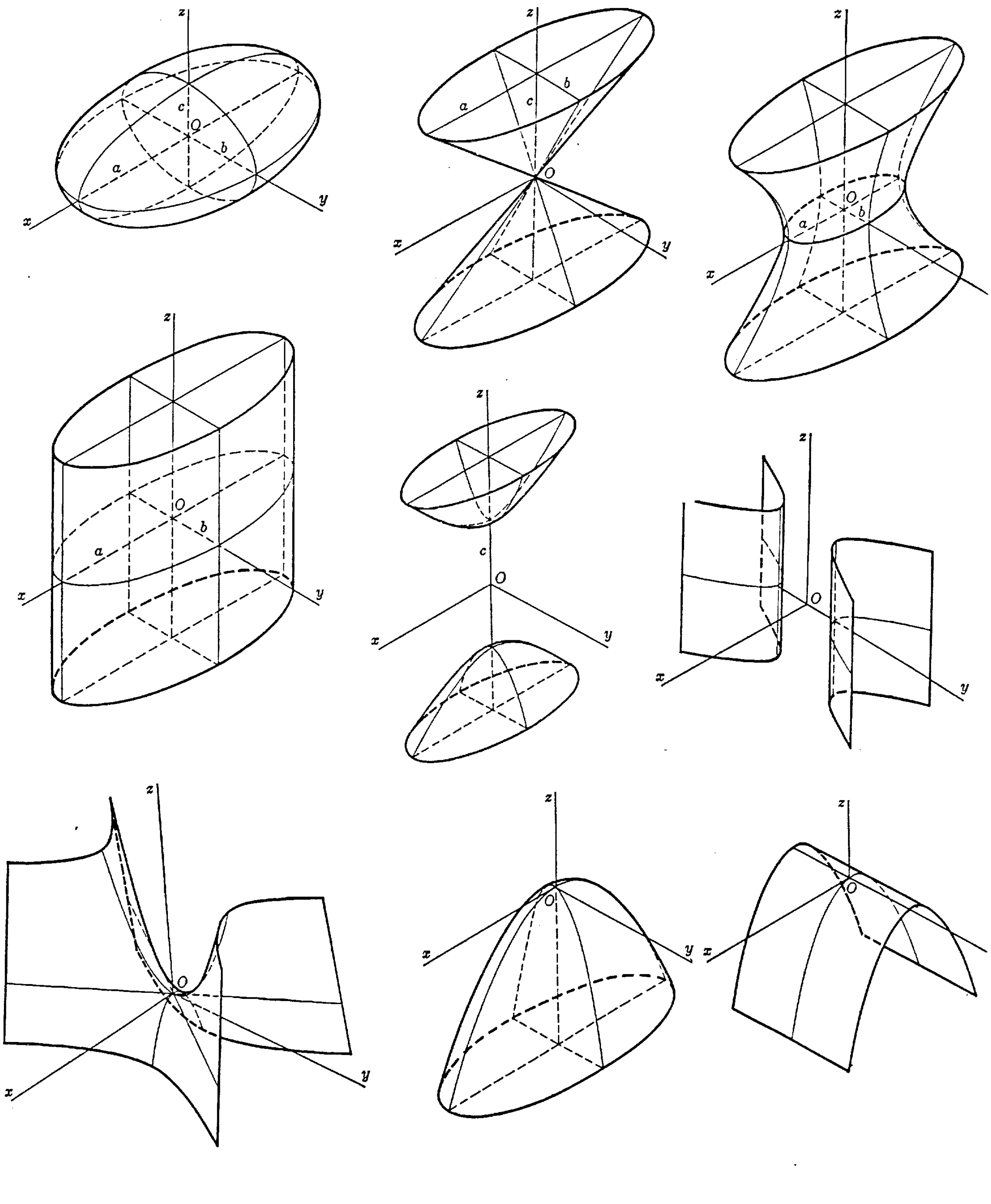


Figure 1. Quadric surfaces from left to right and top to bottom: Ellipsoid, Quadric cone, Hyperboloid of one sheet, Elliptic cylinder, Hyperboloid of two sheets, Hyperbolic cylinder, Hyperbolic paraboloid, Elliptic paraboloid, and Parabolic cylinder.

Recently [9] recognition of 3-D objects based upon their representation as linear combination of 2-D images has been looked into. Transformations such as rotation and translation has been considered for the 3-D objects in terms of the linear combination of a series of 2-D views of the object. Intersections of various 3-D objects with each other, the edges obtained thereby, has been [10] looked into as means of obtaining a surface description vector (SDV) graph for representing natural quadric objects. This research proposes an optimal sensing strategy of a sensor system engaged in the recognition and localization of 3-D quadric objects.

3. THEORY

Any quadric surface can be represented in terms of a second degree polynomial of variables **x**, **y**, and **z**, such that

$$F(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2px + 2qy + 2rz + d = 0 \tag{1}$$

Let (x,y,z) describe the coordinates of any point in our coordinate system. Consider a rotation of α about **z** axis, then the old coordinates in terms of the new one are represented as

$$x = x'\cos\alpha + y'\sin\alpha \tag{2}$$

$$y = -x'\sin\alpha + y'\cos\alpha \tag{3}$$

i.e., the rotation matrix is

$$R_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Next consider a rotation about the $\mathbf{x}'$ axis by an angle β of the same point. The resultant coordinates and the old coordinates are now related by the following equations

$$z = z'\cos\beta - y''\sin\beta \tag{5}$$

$$y' = z'\sin\beta + y''\cos\beta \tag{6}$$

whereby the rotation matrix is

$$R_\beta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \tag{7}$$

And finally consider a rotation about the y' axis by an angle γ, then

$$z' = z''\cos\gamma + x''\sin\gamma \tag{8}$$

$$x' = z''\sin\gamma - x''\cos\gamma \tag{9}$$

The rotation matrix for the above process was

$$R_\gamma = \begin{bmatrix} \cos\gamma & 0 & \sin\gamma \\ 0 & 1 & 0 \\ -\sin\gamma & 0 & \cos\gamma \end{bmatrix}$$

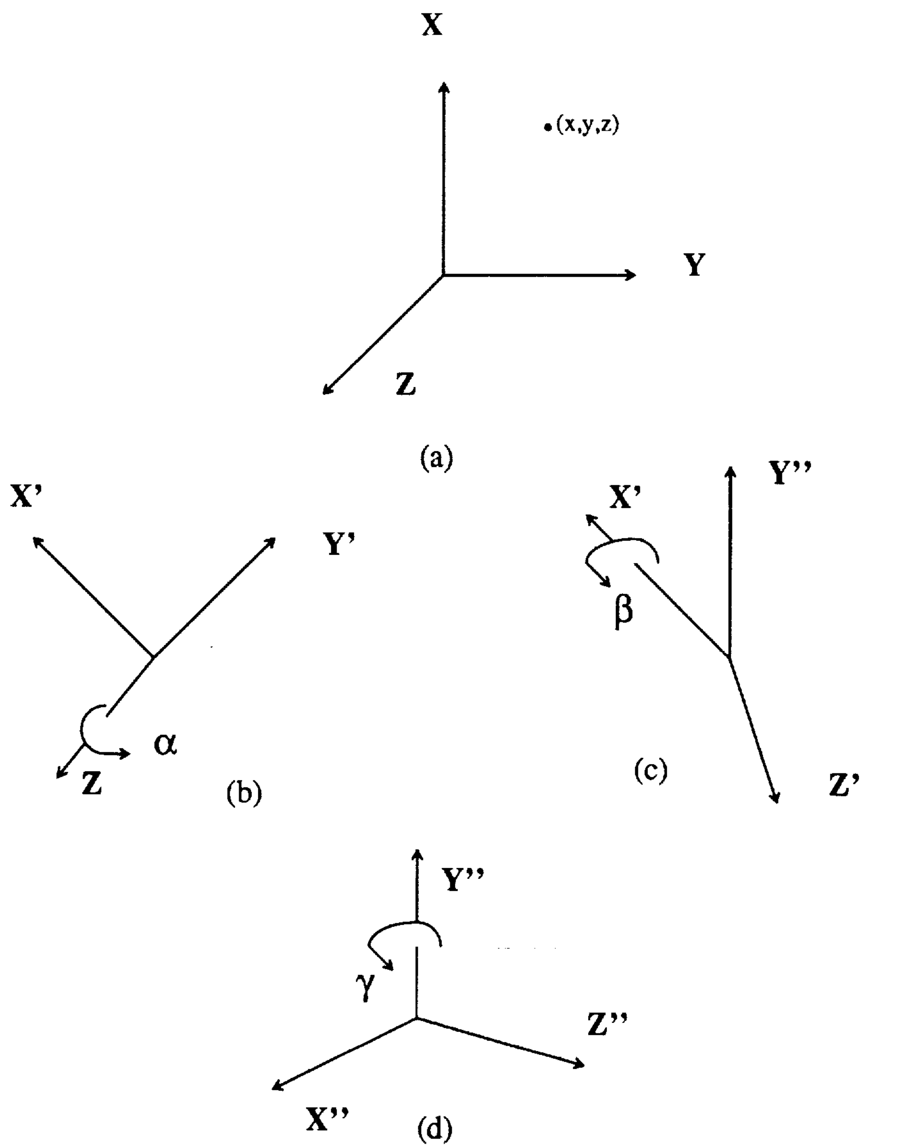
Figure (2) illustrates the behavior of a particular point (x,y,z) with respect the various rotations described above.

Using the above set of equations and solving for x, y, and z in terms of x'', y'', and z'', yields

$$x = -\,x''(\cos\alpha\cos\gamma + \sin\alpha\sin\beta\sin\gamma) + y''\sin\alpha\cos\beta + z''(\sin\gamma\cos\alpha + \cos\gamma\sin\alpha\sin\beta)$$

$$y = x''(\cos\gamma\sin\alpha - \sin\gamma\sin\beta\cos\alpha) + y''\cos\beta\cos\alpha + z''(\cos\gamma\sin\beta\cos\alpha - \sin\gamma\sin\alpha)$$

$$z = -x''\sin\gamma\cos\beta - y''\sin\beta + z''\cos\gamma\cos\beta$$



Figures 2(a), (b), (c), and (d) refer to the coordinate system initially, after the first rotation, the second rotation, and subsequently the third rotation, respectively.

After substituting these new **x, y,** and **z** coordinates in equation (1), we get an entire set of new coefficients for x^2, y^2, z^2, **yz, xz, xy, x, y,** and **z**. These new coefficients are as listed below :

$$A' = \cos^2\gamma\,[a\cdot\cos^2\alpha + b\cdot\sin^2\alpha] + \sin^2\beta\sin^2\gamma\,[a\cdot\sin^2\alpha + b\cdot\cos^2\alpha] + 2\sin\alpha\sin\beta\sin\gamma\cos\alpha\cos\gamma\,(a - b) \tag{10}$$
$$+ c\cdot\sin^2\gamma\cos^2\beta - \sin 2\alpha\,[h\cdot\sin^2\beta\sin^2\gamma] - \sin 2\gamma\,[f\cdot\sin\alpha\cos\beta + g\cdot\cos\alpha\cos\beta + h\cdot\cos^2\alpha\sin\beta - h\cdot\sin\beta\sin^2\alpha\,]$$
$$+ \sin 2\beta\sin^2\gamma\,(f\cdot\cos\alpha - g\cdot\sin\alpha) + h\cdot\sin 2\alpha\cos^2\gamma$$

$$B' = (a\cdot\sin^2\alpha + b\cdot\cos^2\alpha)\cos^2\beta + c\cdot\sin^2\beta + \sin 2\beta\,[-f\cdot\cos\alpha - g\cdot\sin\alpha\,] + h\cdot\sin 2\alpha\cos^2\beta \tag{11}$$

$$C' = \sin^2\gamma\,(a\cdot\cos^2\alpha + b\cdot\sin^2\alpha) + (a\cdot\sin^2\alpha + b\cdot\cos^2\alpha)\cos^2\gamma\sin^2\beta + 2\sin\alpha\sin\beta\sin\gamma\cos\alpha\cos\gamma\,(a - b) \tag{12}$$
$$+ c\cdot\cos^2\gamma\cos^2\beta + \sin 2\alpha\,[h\cdot\cos^2\gamma\sin^2\beta - h\cdot\sin^2\gamma] + \cos^2\gamma\sin 2\beta\,[f\cdot\cos\alpha + g\cdot\sin\alpha]$$
$$+ \sin 2\gamma\,[-f\cdot\sin\alpha\cos\beta + g\cdot\cos\alpha\cos\beta + h\cdot\cos 2\alpha\sin\beta]$$

$$2F' = \Big[(b\cdot\cos^2\alpha + a\cdot\sin^2\alpha + h\cdot\sin 2\alpha - c)\sin 2\beta + (2g\cdot\sin\alpha + 2f\cdot\cos\alpha)\cos 2\beta\Big]\cos\gamma$$
$$+ \Big[((a - b)\sin 2\alpha + 2h\cdot\cos 2\alpha)\cos\beta - (2g\cdot\cos\alpha - 2f\cdot\sin\alpha)\sin\beta\Big]\sin\gamma \tag{13}$$

$$2G' = \sin 2\gamma\Big[-\cos^2\alpha(a + b\cdot\sin^2\beta) - \sin^2\alpha(a\cdot\sin^2\beta + b) - c\cdot\cos^2\beta \tag{14}$$
$$- \sin\beta\cos\beta(f\cdot\cos\alpha + g\cdot\sin\alpha) + h\cdot\sin\alpha\cos\alpha\cos\beta$$
$$+ \sin 2\alpha(a + b)\sin\beta + 2\cos\beta\cos^2\gamma(f\cdot\sin\alpha - g\cdot\cos\alpha) + 2h\cdot\sin\beta(\sin^2\alpha\sin 2\gamma - \cos^2\alpha\cos^2\gamma)$$

$$2H' = \sin 2\alpha\Big[\cos\alpha\cos\beta(b - a) - h\cdot\sin\beta\sin\gamma\cos\beta\Big] + \sin 2\beta\sin\gamma(a\cdot\sin^2\alpha - b\cdot\cos^2\alpha + c) \tag{15}$$
$$+ \cos\gamma\sin\beta(2g\cdot\cos\alpha - 2f\cdot\sin\alpha) + \sin^2\beta\sin\gamma(2g\cdot\sin\alpha + 2f\cdot\cos\alpha) - 2h\cdot\cos^2\alpha\cos\gamma\cos\beta$$

$$2P' = 2\cos\gamma\,[-p\cdot\cos\alpha + q\cdot\sin\alpha] - 2\sin\beta\sin\gamma\,[p\cdot\sin\alpha + q\cdot\cos\alpha] - 2r\cdot\sin\gamma\cos\beta \tag{16}$$

$$2Q' = 2\cos\beta\,[p\cdot\sin\alpha + q\cdot\cos\alpha] - 2r\cdot\sin\beta \tag{17}$$

$$2R' = 2\cos\gamma\sin\beta\,[p\cdot\sin\alpha + q\cdot\cos\alpha] + 2\sin\gamma\,[p\cdot\cos\alpha - q\cdot\sin\alpha] + 2r\cdot\cos\gamma\cos\beta \tag{18}$$

$$D' = D$$

As seen from above, except for the constant **D**, all of the other coefficients are affected by the rotation of α, β, and γ.

3.1. Scheme

The product terms **yz, xz,** and **xy** in f(x,y,z) above, denote the rotation terms which are to be eliminated. Elimination of all these rotation terms will place the 3-D surface on a coordinate system plane parallel to our coordinate system.

At first sight, this problem appears quite simple. Eliminating each term, i.e., by rotating the surface about the origin in a particular orientation by a suitable angle θ, then eliminating the second term and then the next term. But that's not the case. In the presence of a single rotation term i.e., if the equation is in the form

$$F(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2px + 2qy + 2rz + d = 0,$$

equation of the trace of the above surface in **yz** plane is obtained by setting $\mathbf{x = 0}$, so a rotation about the origin in the **yz** plane by an angle θ will eliminate the term **yz** from the above equation.

However, in the presence of two or more rotation terms, while trying to eliminate a second rotation term, it is seen that the rotation term eliminated first, reappears and thereby at any given instance there will be at least two rotation terms available. The approach we have proposed is an iterative process, whereby at each stage the object is rotated in each of the directions **x, y,** and **z** sequentially and the process is carried out until all the product terms fall by, i.e., the coefficients **f, g,** and **h** converge to zero. Since our aim is to eliminate the rotation terms **xy, yz,** and **xz,** let's now exclusively consider the coefficients of these rotation terms, namely **F, G,** and **H** evaluated above. In an iterative procedure we will be able to eliminate all of the product terms. For e.g., we wish to eliminate the term **xy.** Then, by a specific rotation of α about the

z axis, we will be able to accomplish our goal. However, while carrying out this process, the orientation of the object about the two planes **yz** and **xz** , i.e., the angles the object made with these two planes have been disturbed. Now if we wish to eliminate the **yz** term, the object has to be rotated about the **x** axis by an angle β. However, in this instance, while carrying out the process, the already missing **xy** term reappears though the magnitude of its present orientation has been reduced. Hence by carrying out the above process in an iterative fashion, there comes an instance when all the coefficients of the product terms converge to zero.

Let us consider the equations (13), (14), and (15) respectively. Let us eliminate the coefficient **h,** i.e, the **xy** term in step **1.** This can be accomplished by rotating the object about the **z** axis by an angle α, whereas $\beta = \gamma = 0$. Under these circumstances the new coefficients look like as shown below:

$$2f_{11} = 2g \cdot \sin\alpha_1 + 2f \cdot \cos\alpha_1$$

$$2g_{11} = 2g \cdot \cos\alpha_1 - 2f \cdot \sin\alpha_1$$

$$2h_{11} = (a - b)\sin 2\alpha_1 + 2h \cdot \cos 2\alpha_1 = 0 \quad \text{where} \quad \cot 2\alpha_1 = b - \frac{a}{2h}$$

As seen above the coefficient **h** has been forced to 0. The most significant bit of the subscript refers to the iteration number , whereas the least significant bit of the subscript reflects the number of times the object has been rotated by a specific angle. In the above case the LSB of **1** refers to the first instance where the object has been rotated by an angle $\boldsymbol{\alpha}$. The remaining coefficients **a, b, c, p, q,** and **r** also reflect changes brought about by the above rotation. The new coefficients look as shown below:

$$a_{11} = a \cdot \cos^2\alpha_1 + b \cdot \sin^2\alpha_1 - 2h \cdot \sin\alpha_1 \cos\alpha_1$$

$$b_{11} = b\cdot\cos^2\alpha_1 + a\cdot\sin^2\alpha_1 + 2h\cdot\sin\alpha_1\cos\alpha_1$$

$$c_{11} = c$$

$$2p_{11} = 2p\cdot\cos\alpha_1 - 2q\cdot\sin\alpha_1$$

$$2q_{11} = 2p\cdot\sin\alpha_1 = 2q\cdot\cos\alpha_1$$

$$2r_{11} = 2r$$

The new quadric equation now has a look as shown below:

$$F(x,y,z) = a_{11}x^2 + b_{11}y^2 + c_{11}z^2 + 2f_{11}yz + 2g_{11}xz + 2p_{11}x + 2q_{11}y + 2r_{11}z + d = 0$$

Consider the second step wherein the coefficient corresponding to the **yz** term is forced to zero. In this particular case, the object has to be rotated by an angle $\boldsymbol{\beta}$ about the **x** axis, whereas $\boldsymbol{\alpha} = \boldsymbol{\gamma} = \mathbf{0}$. Under these circumstances, the new rotation coefficients (signifying the product terms) becomes

$$2f_{12} = (b_{12} - c_{12})\sin2\beta_1 + 2f_{11}\cdot\cos2\beta_1 = 0 \quad \text{where} \quad \cot2\beta_1 = \frac{c_{11} - b_{11}}{2}f_{11}$$

$$2g_{12} = 2g_{11}\cdot\cos\beta_1$$

$$2h_{12} = -2g_{11}\cdot\sin\beta_1$$

At the same time the other coefficients become

$$a_{12} = a_{11}$$

$$b_{12} = c_{11}\cdot\sin^2\beta_1 + b_{11}\cdot\cos^2\beta_1 - 2f_{11}\cdot\sin\beta_1\cos\beta_1$$

$$c_{12} = b_{11}\cdot\sin^2\beta_1 + c_{11}\cdot\cos^2\beta_1 + 2f_{11}\cdot\sin\beta_1\cos\beta_1$$

$$2p_{12} = 2p_{11}$$

$$2q_{12} = 2q_{11}\cdot\cos\beta_1 - 2r_{11}\cdot\sin\beta_1$$

$$2r_{12} = 2q_{11}\cdot\sin\beta_1 + 2r_{11}\cdot\cos\beta_1$$

The new quadric equation as before looks like as shown below:

$$F(x,y,z) = a_{12}x^2 + b_{12}y^2 + c_{12}z^2 + 2g_{12}xz + 2h_{12}xy + 2p_{12}x + 2q_{12}y + 2r_{12}z + d = 0$$

In the final step of the initial iteration, the coefficient corresponding to the **xz** term is forced to zero. In this case, the object is to be rotated by an angle $\boldsymbol{\gamma}$ about the **y** axis, whereas $\boldsymbol{\alpha} = \boldsymbol{\beta} = \mathbf{0}$. Under these circumstances, the new rotation coefficients become

$$2f_{13} = 2h_{12}\cdot\sin\gamma_1 = -2g_{11}\cdot\sin\beta_1\sin\gamma_1$$

$$2g_{13} = (a_{13} - c_{13})\sin2\gamma_1 + (2g_{11}\cdot\cos\alpha_1 - f_{11}\cdot\sin\alpha_1)\cos\beta_1\cos2\gamma_1 = 0$$

$$\text{where} \quad \cot2\gamma_1 = \frac{c_{12} - a_{12}}{2}g_{12}$$

$$2h_{13} = 2h_{12}\cdot\cos\gamma_1 = -2g_{11}\cdot\sin\beta_1\cos\gamma_1$$

Let's now carefully analyze the coefficients of **xy, yz,** and **zx** obtained in the final step of the first iteration. Consider for instance the coefficient corresponding to the **yz** term. It is observed that while proceeding from one step to the other, the new coefficients are getting multiplied by the **sine** or **cosine** of the concerned angle. This implies that in every succeeding steps, these coefficients are decreasing in their magnitude. To justify the above statement, let us now consider all the coefficients obtained in the second iteration.

At the end of stage **1** of the second iteration, the rotation coefficients become

$$2f_{21} = 2f_{13}\cdot\cos\alpha_2 = -2g_{11}\cdot\sin\beta_1\sin\gamma_1\cos\alpha_2$$

$$2g_{21} = -2f_{13}\cdot\sin\alpha_2 = 2g_{11}\cdot\sin\beta_1\sin\gamma_1\sin\alpha_2$$

$$2h_{21} = 0 \quad \text{where} \quad \cot 2\alpha_2 = \frac{b_{13} - a_{13}}{2} h_{13}$$

At the end of the second stage of the second iteration, the rotation coefficients become

$$2f_{22} = 0 \quad \text{where} \quad \cot 2\beta_2 = \frac{c_{21} - b_{21}}{2} f_{21}$$

$$2g_{22} = 2g_{11}\cdot\sin\beta_1\sin\gamma_1\sin\alpha_2\cos\beta_2$$

$$2h_{22} = -2g_{11}\cdot\sin\beta_1\sin\gamma_1\sin\alpha_2\sin\beta_2$$

Similarly at the end of the final stage of the second iteration, the rotation coefficients reduce to

$$2f_{23} = -2g_{11}\cdot\sin\beta_1\sin\gamma_1\sin\alpha_2\sin\beta_2\sin\gamma_2$$

$$2g_{23} = 0 \quad \text{where} \quad \cot 2\alpha_2 = \frac{b_{13} - a_{13}}{2} h_{13}$$

$$2h_{23} = -2g_{11}\cdot\sin\beta_1\sin\gamma_1\sin\alpha_2\sin\beta_2\cos\gamma_2$$

α_2, β_2,and γ_2 are the respective rotation angles along the **z, x,** and **y** axes in the second iteration. Hence it is observed with each iteration that the rotation coefficients get smaller and smaller in magnitude and eventually drop out.

We are now in a position to formulate a rotation matrix whose elements correspond to the directional cosines of the **x, y,** and **z** axes of the rotated object.

$$\text{Rotation Matrix} = R_\gamma\, R_\beta\, R_\alpha \tag{19}$$

where

$$R_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_\beta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix}$$

$$R_\gamma = \begin{bmatrix} \cos\gamma & 0 & \sin\gamma \\ 0 & 1 & 0 \\ -\sin\gamma & 0 & \cos\gamma \end{bmatrix}$$

Subsequently,

$$R_\gamma R_\beta R_\alpha = \begin{bmatrix} \cos\alpha\cos\gamma + \sin\alpha\sin\beta\sin\gamma & \cos\gamma\sin\alpha - \sin\gamma\sin\beta\cos\alpha & \sin\gamma\cos\beta \\ -\cos\beta\sin\alpha & \cos\beta\cos\alpha & \cos\beta \\ -\sin\gamma\cos\alpha + \cos\gamma\sin\alpha\sin\beta & -\sin\alpha\sin\gamma - \cos\gamma\sin\beta\cos\alpha & \cos\beta\cos\gamma \end{bmatrix} \tag{20}$$

where

$\alpha^* = \sum_{i=1}^{n} \alpha_i$, $\beta^* = \sum_{i=1}^{n} \beta_i$, and $\gamma^* = \sum_{i=1}^{n} \gamma_i$.

n corresponds to the iteration where all the rotation terms go to zero.

Once the rotation terms, i.e., **xy, yz,** and **xz** are eliminated, the 3-D surface has the representation of

$$F(x,y,z) = Ax^2 + By^2 + Cz^2 + 2Px + 2Qy + 2Rz + D = 0 \tag{21}$$

where **A, B, C, P, Q,** and **R** are the coefficients evolved after the elimination of the rotation terms.

A natural question to ask is the following : " Can the terms of the first degree be

* proved numerically and experimentally.

eliminated by means of a translation ?"The answer is, "sometimes they can and sometimes they cannot". For the case, where the term can be eliminated, is supported by the following theorem [11].

3.2. Translation of the rotated object

Theorem: The terms of the first degree of an equation of a quadric surface can be eliminated by means of a translation if and only if the surface has a center, in which case the first degree terms are eliminated if and only if the new origin is a center[11].

Since all the 3-D object we are considering do have centers, we do not have to worry about the second case. The method of completing squares is the easiest to determine the coordinates of the new origin.

Consider equation (21), grouping the like terms

$$Ax^2 + 2Px + By^2 + 2Qy + Cz^2 + 2Rz + D = 0 \quad \Rightarrow$$

$$A\left(x^2 + 2P\frac{x}{A}\right) + B\left(y^2 + 2Q\frac{y}{B}\right) + C\left(z^2 + 2R\frac{z}{C}\right) + D = 0$$

Upon completing squares, we get

$$A\left(x + \frac{P}{A}\right)^2 + B\left(y + \frac{Q}{B}\right)^2 + C\left(z + \frac{R}{C}\right)^2 + D - \left(\frac{P^2}{A} + \frac{Q^2}{B} + \frac{R^2}{C}\right) = 0 \qquad (22)$$

where -P/A, -Q/B, and -R/C are the coordinates of the new origin.

4. CONCLUSIONS

In this research we have proposed a method to determine the pose and orientation of a natural quadric surface from its range image and subsequently eliminate all of these rotation parameters.

Once the rotation and the translation procedures accomplish their objective of placing a quadric surface in a desired coordinate system, we will be in a situation to implement the necessary recognition process. We wish to extend upon our previous work [7] to the various quadric surfaces as mentioned before. Experiments for the alignment algorithm were performed on range data of circular cylinder rotated in space. The results obtained were very promising. Range data of quadric cones, parabolic cylinders, and hyperboloids are presently being investigated.

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